UNIVERSIDADE FEDERAL DE PERNAMBUCO

CCEN – DEPARTAMENTO DE MATEMÁTICA – ÁREA II

CÁLCULO DIFERENCIAL E INTEGRAL 3

LISTA DE EXERCÍCIOS

SÉRIES DE POTÊNCIAS

1. Determine o intervalo de convergência das seguintes séries:

$a)\ \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n+1}$ $\quad b)\ \sum_{n=1}^{\infty}\frac{x^n}{n\,3^n}$ $\quad c)\ \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

$d)\ \sum_{n=2}^{\infty}(-1)^n\frac{x^n}{4^n\,\ln n}$ $\quad e)\ \sum_{n=0}^{\infty}\frac{n\,(x-4)^n}{n^3+1}$ $\quad f)\ \sum_{n=1}^{\infty}\frac{n^2x^n}{2\cdot 4\cdot 6\cdots(2n)}$

$g)\ \sum_{n=0}^{\infty}\frac{n\,x^n}{2^n}$ $\quad h)\ \sum_{n=0}^{\infty}\frac{(x+1)^n}{n^2}$ $\quad i)\ \sum_{n=0}^{\infty}n\,x^n$

$j)\ \sum_{n=1}^{\infty}\frac{(x-1)^n}{n}$ $\quad k)\ \sum_{n=1}^{\infty}\frac{2n\,x^n}{n!}$ $\quad l)\ \sum_{n=1}^{\infty}\frac{(x-1)^n}{n}$

$m)\ \sum_{n=0}^{\infty}\frac{(4x+1)^n}{n^2}$ $\quad n)\ \sum_{n=1}^{\infty}\frac{(n!)^2x^n}{2n!}$ $\quad o)\ \sum_{n=0}^{\infty}\frac{n!\,x^n}{n^n}$

2. Determine a representação em série de potências das seguintes funções e identifique seu intervalo de convergência:

$a)\ f(x)=\frac{1}{1+x}$ $\quad b)\ f(x)=\frac{1}{1+x^2}$ $\quad c)\ f(x)=\frac{1}{1+9x^2}$

$d)\ f(x)=\frac{x^2}{1+9x^2}$ $\quad e)\ f(x)=\frac{x}{1+4x}$ $\quad f)\ f(x)=\arctan x$

$g)\ f(x)=\ln(1+x)$ $\quad h)\ f(x)=\ln(1-x)$ $\quad i)\ f(x)=\ln\frac{1+x}{1-x}$

3. Calcule a soma das séries identificando a função para a qual a série converge, e o respectivo intervalo de convergência:

$a)\ \sum_{n=0}^{\infty}(-1)^n\,x^n$ $\quad b)\ \sum_{n=1}^{\infty}n\,x^n$ $\quad c)\ \sum_{n=1}^{\infty}\frac{x^n}{n}$

$d)\ \sum_{n=1}^{\infty}n\,x^{n-1}$ $\quad e)\ \sum_{n=1}^{\infty}\frac{x^n}{n}$ $\quad f)\ \sum_{n=2}^{\infty}n(n-1)\,x^n$

4. Usando a questão anterior, calcule as seguintes somas:

$a)\ \sum_{n=1}^{\infty}\frac{n}{2^n}$ $\quad b)\ \sum_{n=2}^{\infty}\frac{n^2}{2^n}$ $\quad c)\ \sum_{n=2}^{\infty}\frac{n^2-n}{2^n}$ $\quad d)\ \sum_{n=2}^{\infty}\frac{n}{4^n}$

Respostas:

$1a)\quad -1 < x \leq 1$	$1b)\quad -3 \leq x < 3$	$1c)\quad \mathbb{R}$	$1d)\quad -4 < x \leq 4$
$1e)\quad 3 \leq x \leq 5$	$1f)\quad \mathbb{R}$	$1g)\quad -2 < x < 2$	$1h)\quad -2 \leq x \leq 0$
$1i)\quad -1 < x < 1$	$1j)\quad 0 \leq x < 2$	$1k)\quad \mathbb{R}$	$1l)\quad 0 \leq x < 2$
$1m)\quad \left[-\frac{1}{2}, 0\right]$	$1n)\quad \{0\}$	$1o)\quad -e < x < e$	

$2a)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n x^n,\ |x| < 1$ $\qquad 2b)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n},\ |x| < 1$

$2c)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n 9^n x^{2n},\ |x| < \frac{1}{3}$ $\qquad 2d)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n 9^n x^{2n+2},\ |x| < \frac{1}{3}$

$2e)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n 4^n x^{n+1},\ |x| < \frac{1}{4}$ $\qquad 2f)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2n+1},\ |x| \leq 1$

$2g)\quad f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1},\ |x| \leq 1$ $\qquad 2h)\quad f(x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1},\ |x| \leq 1$

$2i)\quad f(x) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1},\ |x| \leq 1$

$3a)\quad f(x) = \frac{1}{1+x},\ |x| < 1$ $\qquad 3b)\quad f(x) = \frac{x}{(1-x)^2},\ |x| < 1$ $\qquad 3c)\quad f(x) = -\ln(1-x),\ -1 \leq x < 1$

$3d)\quad f(x) = \frac{1}{(1-x)^2},\ |x| < 1$ $\qquad 3e)\quad f(x) = -\ln(1-x),\ -1 \leq x < 1$ $\qquad 3f)\quad f(x) = \frac{2x^2}{(1-x)^3},\ |x| < 1$

$4a)\quad 2 \qquad 4b)\quad 5 \qquad 4c)\quad 4 \qquad 4d)\quad \frac{7}{36}$