Distribuições de Lévy e vidros de spins em Física Estatística e Fotônica

Ernesto P. Raposo

Departamento de Física

Universidade Federal de Pernambuco

Roteiro

- Parte I: Distribuições de Lévy
- Parte II: Vidros de spin
- Parte III: Aplicações em Física Estatística e Fotônica

- Parte I: Distribuições de Lévy

x_i = face de um dado sorteada na i-ésima jogada

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Variável aleatória $y = (x_1 + \ldots + x_N)/N$

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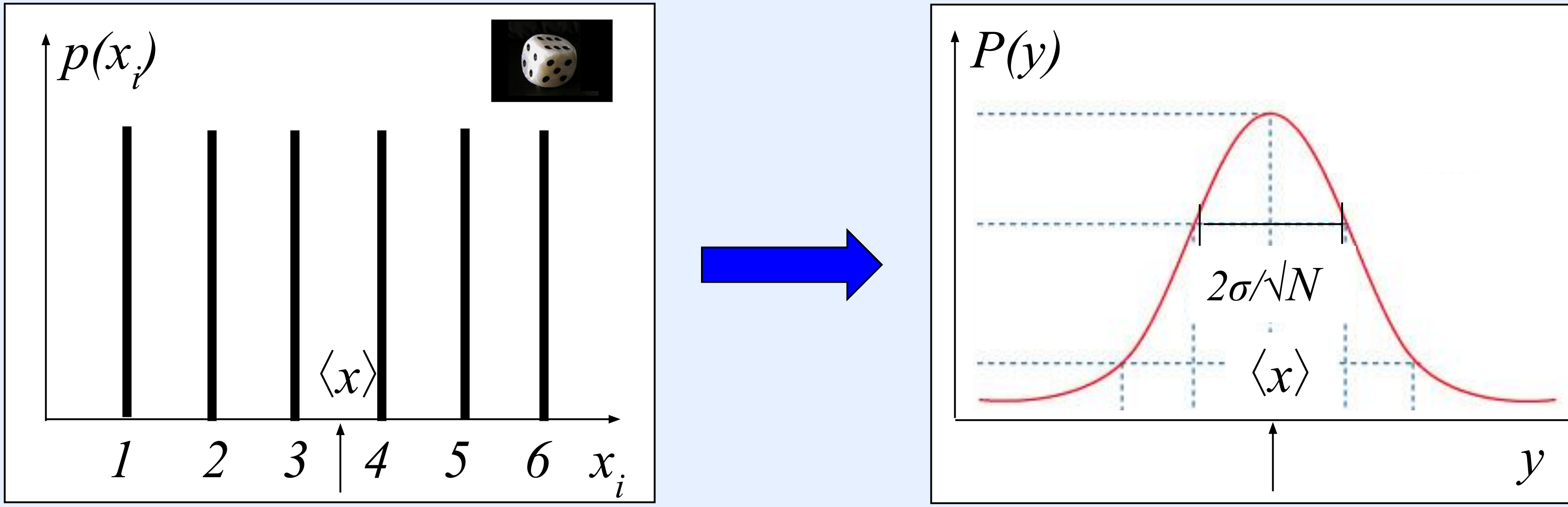
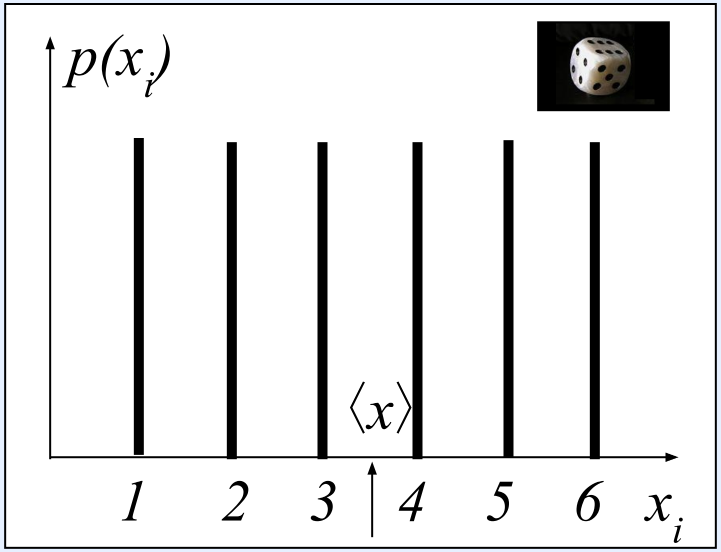


$$p(x_i) = \frac{1}{6}\sum_{f=1}^{6}\delta(x_i - f),\ \forall i$$

$$P(y) \propto \exp\left[-N(y - \langle x \rangle)^2 / 2\sigma^2\right]$$

Porque dist gaussianas são tão comuns?

variável aleatória $y = (x_1 + \ldots + x_N)/N$ possui dist $P(y)$ gaussiana...



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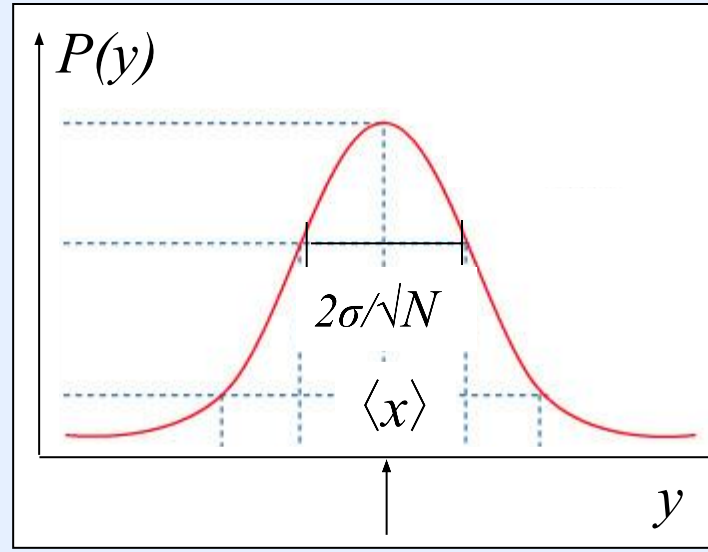


$$P(y) \propto \exp[-N(y-\langle x\rangle)^2/2\sigma^2]$$

e: (i) $N \gg 1$

(ii) x_i estatisticamente independentes

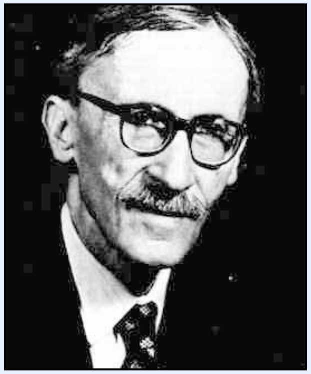
(iii) $p(x_i)$ com segundo momento finito

⇒ **Teorema Central do Limite (TCL)** (Laplace, 1810…)

E se o segundo momento de $p(x)$ divergir?

TCL generalizado → dist de Lévy

(1937)



Paul Lévy

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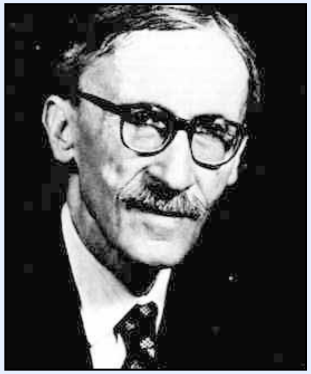
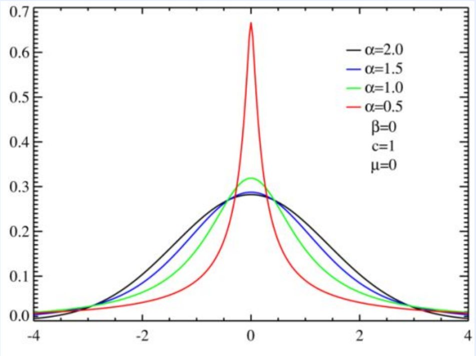


Paul Lévy

$$P(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{P}(k)e^{-iky}dk$$

$$\bar{P}(k) = \exp\{-|ck|^{\alpha}[1 - i\beta\ \mathrm{sgn}(k)\Phi] + ik\nu\}$$

$0 < \alpha < 2$



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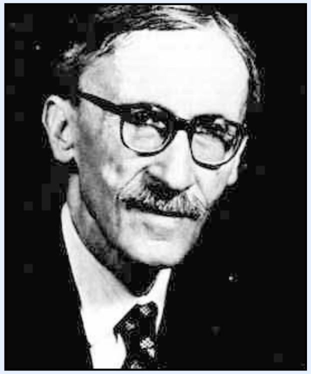

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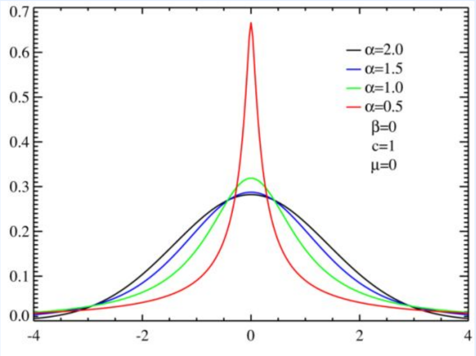
$\alpha = 2$: gaussiana



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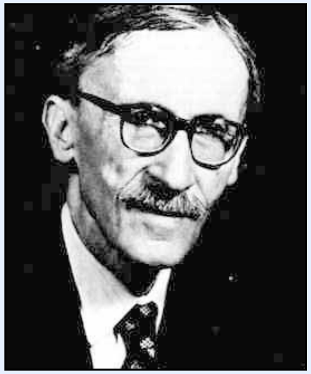

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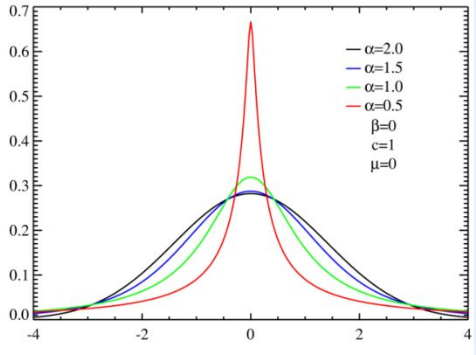

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Expansão: $$P(y) = \frac{A}{|y|^{\alpha+1}} + \dots \quad |y| \gg 1$$ **(lei de potência)**

Ex.: $$p(x) \sim \frac{1}{|x|^{\mu}}$$

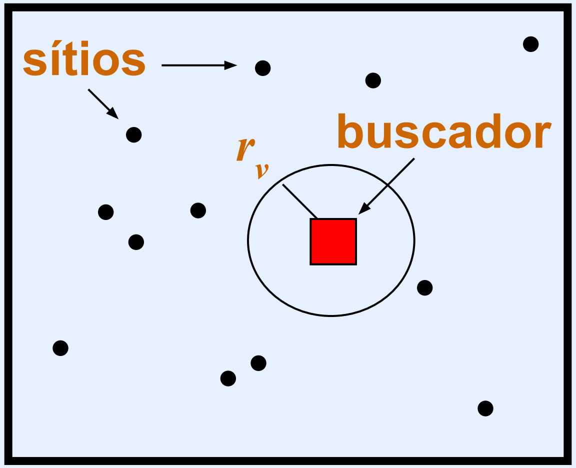
⇒ $\mu \geq 3$ → $P(y)$ **gaussiana (TCL),** $\alpha = 2$

⇒ $1 < \mu < 3$ → $P(y)$ **Lévy (TCLg),** $\alpha = \mu - 1$

- Parte III: Aplicações em Física Estatística e Fotônica

- Buscas aleatórias ("random search walks")

Qual a estratégia mais eficiente que se deve adotar ao se buscar por sítios aleatoriamente distribuídos, quando se tem uma visão limitada (raio r_v) do espaço de busca?



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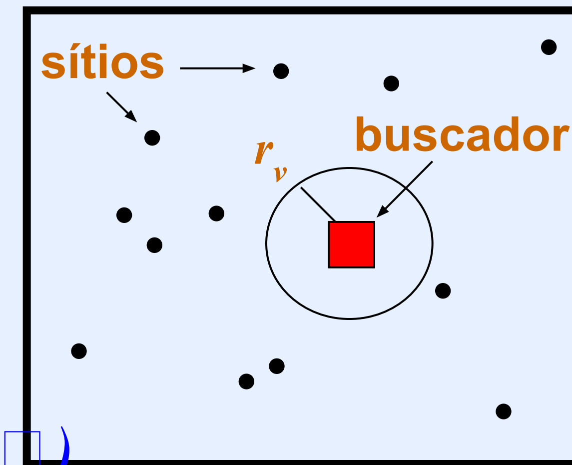
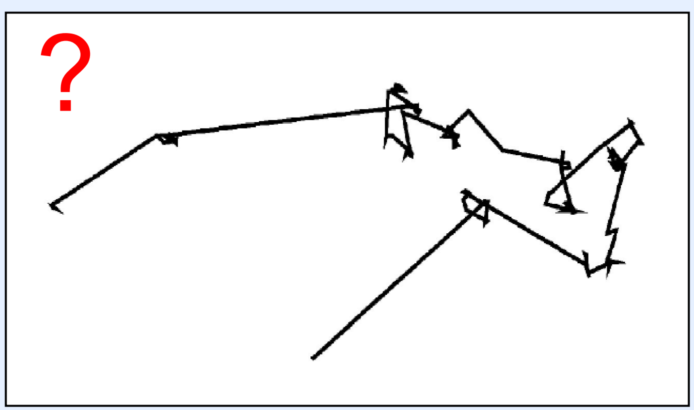
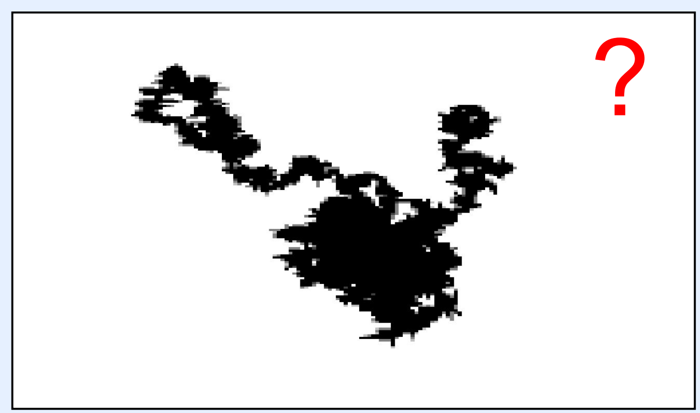


Eficiência da busca:

$$\eta = \frac{\text{número de sítios encontrados}}{\text{distância total percorrida}}$$

Estratégia $\leftrightarrow p(\ell)d\ell$: probabilidade de dar um passo de tamanho $[\ell, \ell + d\ell)$





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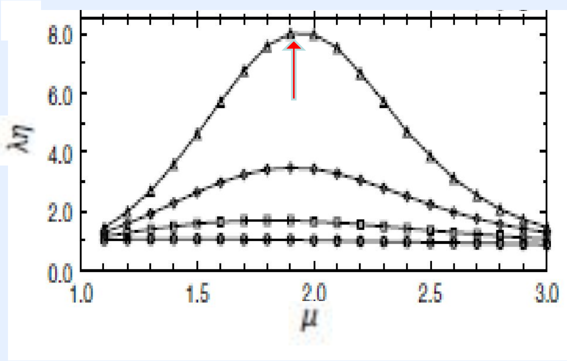

Optimizing the success of random searches

letters to nature

G. M. Viswanathan*†‡, Sergey V. Buldyrev*, Shlomo Havlin*§, M. G. E. da Luz‖¶, E. P. Raposo# & H. Eugene Stanley*

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VOLUME 91, NUMBER 24 — PHYSICAL REVIEW LETTERS — week ending 12 DECEMBER 2003

Dynamical Robustness of Lévy Search Strategies

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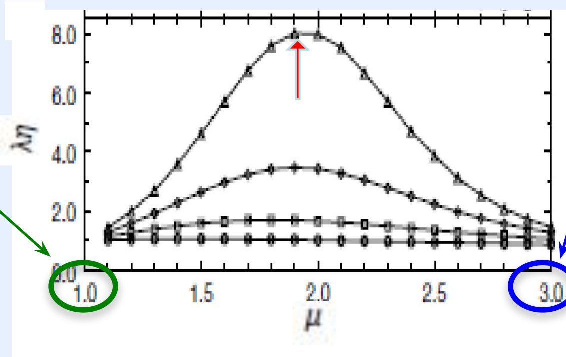
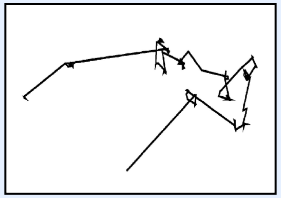
μ → 1: longos passos
acesso a novas regiões



μ → 3: pequenos passos
busca local detalhada



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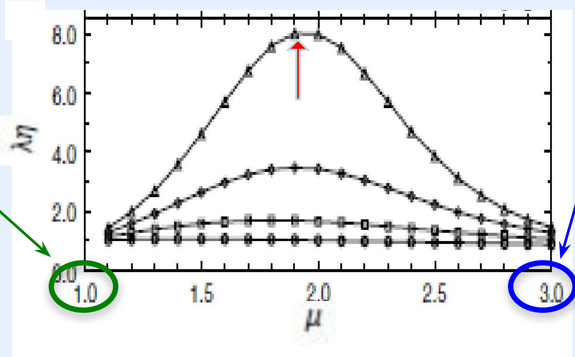

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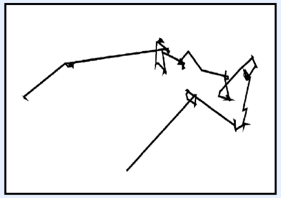

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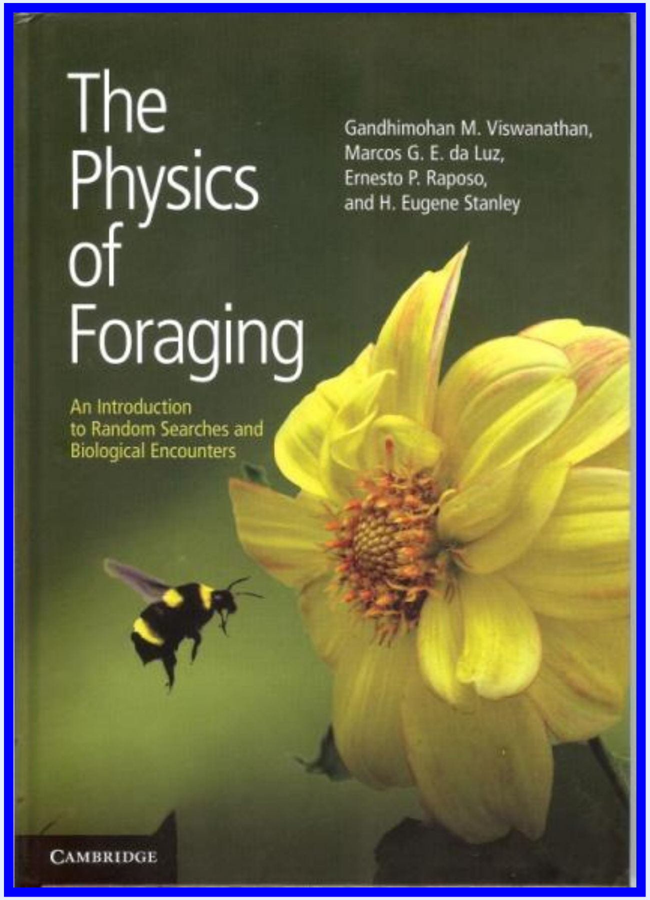
$$\langle l \rangle \approx \frac{\int_{r_v}^{\lambda} l^{1-\mu} dl + \lambda \int_{\lambda}^{\infty} l^{-\mu} dl}{\int_{r_v}^{\infty} l^{-\mu} dl} = \left(\frac{\mu-1}{2-\mu}\right)\left(\frac{\lambda^{2-\mu} - r_v^{2-\mu}}{r_v^{1-\mu}}\right) + \frac{\lambda^{2-\mu}}{r_v^{1-\mu}}$$

$$\mu_{opt} = 2 - 1/[\ln(\lambda/r_v)]^2$$


The Physics of Foraging
Gandhimohan M. Viswanathan,
Marcos G. E. da Luz,
Ernesto P. Raposo,
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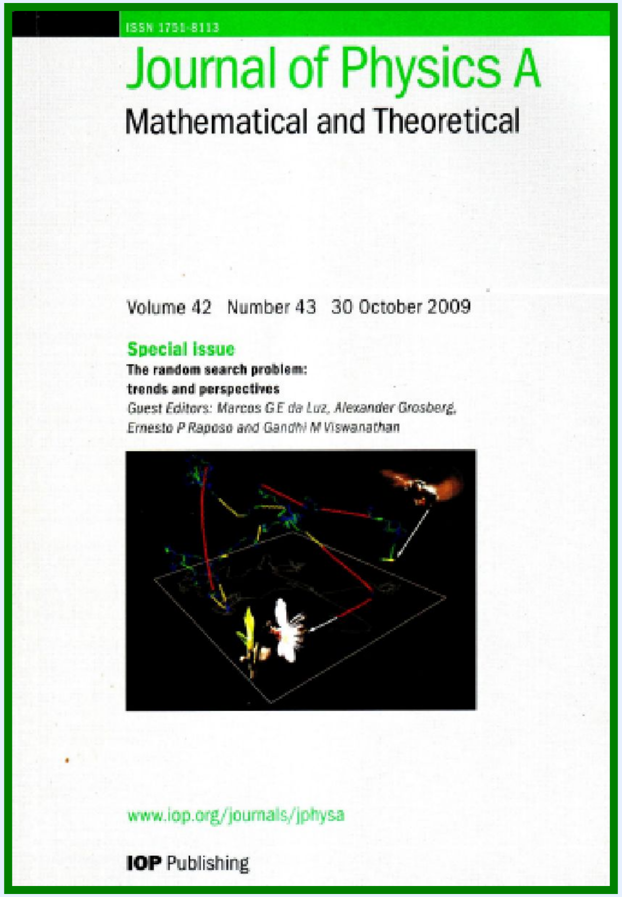
An Introduction to Random Searches and Biological Encounters
CAMBRIDGE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Mathematical and Theoretical
Volume 42 Number 43 30 October 2009
Special Issue
The random search problem: trends and perspectives
Guest Editors: Marcos G E da Luz, Alexander Grosberg, Ernesto P Raposo and Gandhi M Viswanathan
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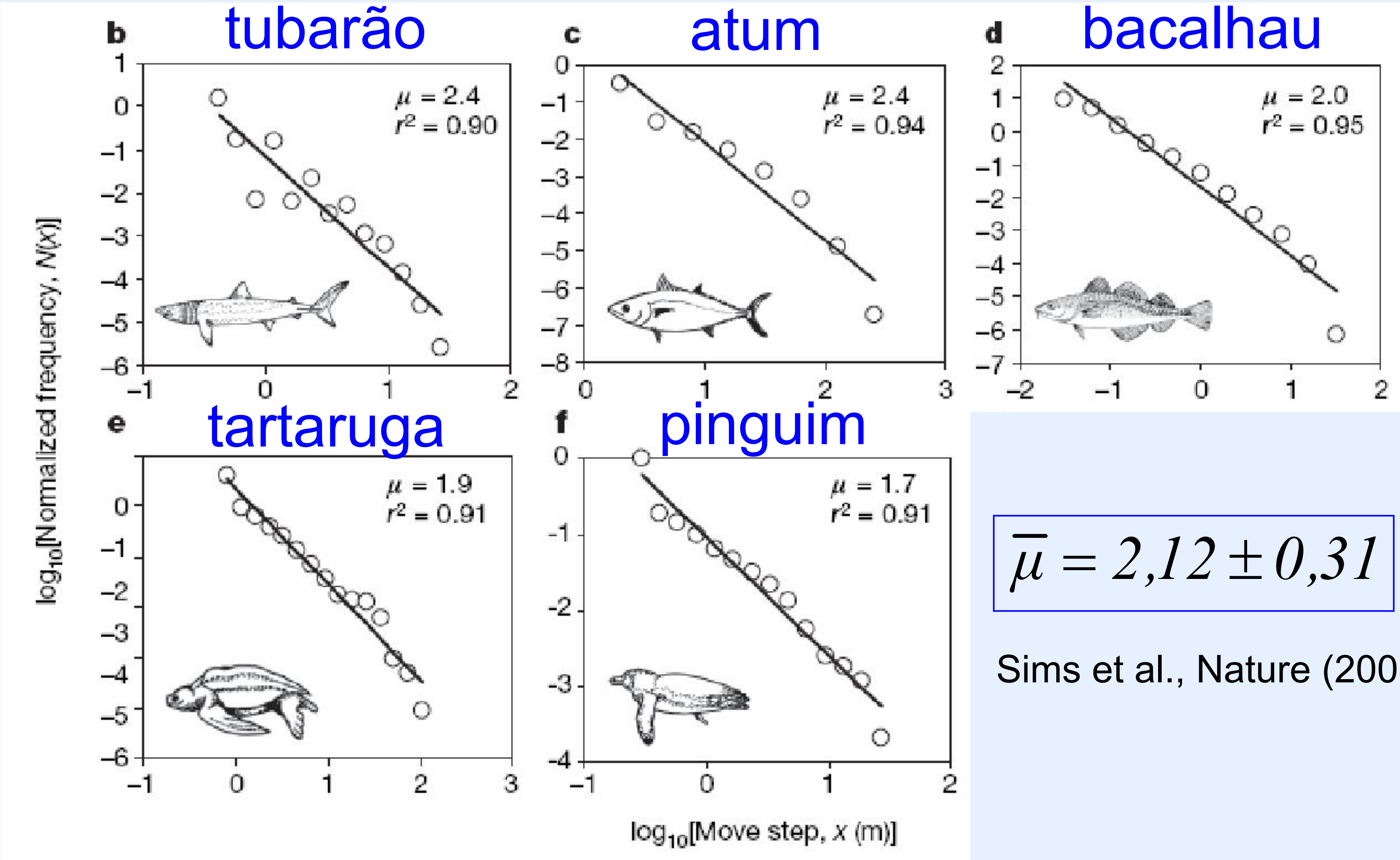
IOP Publishing

Aplicação: animal foraging

Dist de Lévy: $p(\ell) \propto \ell^{-\mu}$

$(1 < \mu < 3)$ (limite assintótico de ℓ grandes) (dist de Pareto)



$$\bar{\mu} = 2{,}12 \pm 0{,}31$$

Sims et al., Nature (2008)...

Aplicação: random lasers

Intensidades (Sharma et al, Fluct. Noise Lett. (2005)) ⇒ Lévy + truncagem exp

PHYSICAL REVIEW A **91**, 043827 (2015)

Analytical solution for the Lévy-like steady-state distribution of intensities in random lasers

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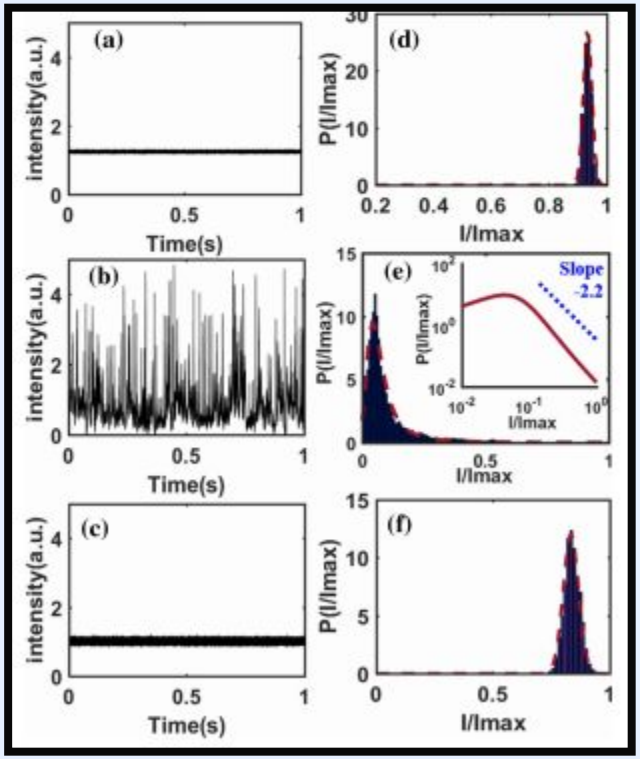

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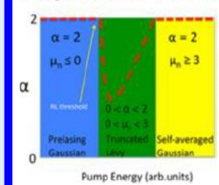

The Levy-like steady-state distributions of emission intensities in random lasers are derived analytically, and it is shown that the Levy exponent alpha can act as a universal identifier of the lasing threshold.

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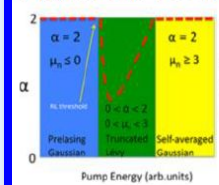

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Modos EM:

$$\mathrm{E}(\mathbf{r},t) = \mathrm{Re}\left\{\sum_k a_k(t)\mathbf{E}_k(\mathbf{r})\exp(-i\omega_k t)\right\}$$

⇓

Eq Langevin:

$$\frac{da_{k_1}}{dt} = -\sum_{k_2'} g^{(2)}_{k_1k_2} a_{k_2} + \frac{1}{2}\sum_{\{k_2k_3k_4\}'} g^{(4)}_{k_1k_2k_3k_4} a_{k_2} a^*_{k_3} a_{k_4} + \eta_{k_1}$$

desordem NL($\chi^{(3)}$) ruído óptico

⇓

Eq Fokker-Planck:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial I_k}\left[\left(-d_k I_k - b_k I_k^2 + 2Q I_k\right) P\right] + 2Q\frac{\partial^2}{\partial I_k^2}\left(I_k^2 P\right)$$

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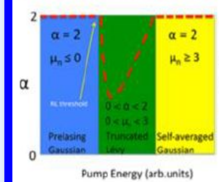

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PHYSICAL REVIEW A **102**, 063515 (2020)

Influence of fifth-order nonlinearities on the statistical fluctuations in emission intensities in a photonic open-cavity complex system

Iván R. R. González,[1] Ernesto P. Raposo,[1,2,3] Sandra J. Carreño,[4] Antônio M. S. Macêdo,[1] Melissa Maldonado,[5] Leonardo de S. Menezes,[5] Anderson S. L. Gomes,[5] and Cid B. de Araújo[5]

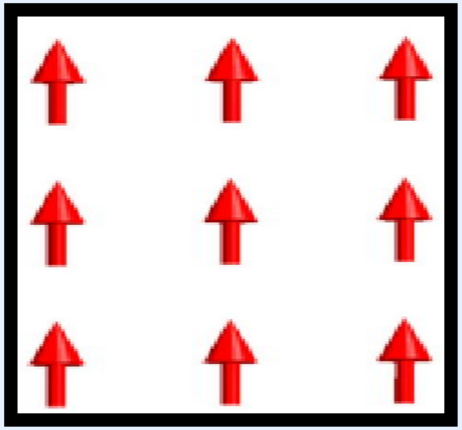
P(I) Lévy-like:

dist Izrailev generalizada

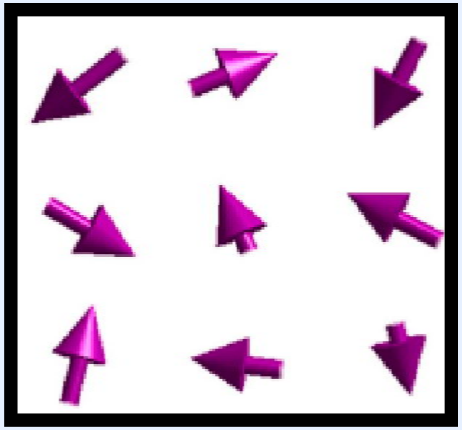
- Parte II: Vidros de spin

Vidros de spins magnéticos ('spin glasses')

Spins 'congelam' em direções aleatórias
a temperaturas $T < T_f$



Ferromagnetismo



Vidro de spins

(Adaptado de S. Sato et al, Symmetry 12, 1910 (2020))

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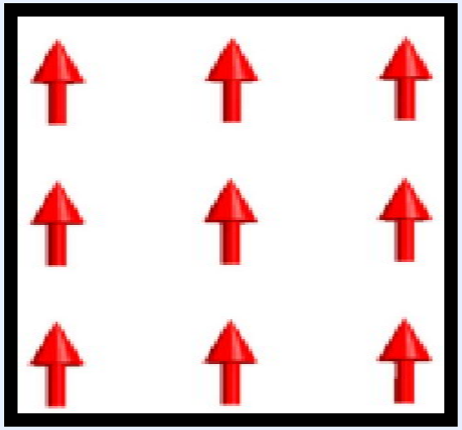

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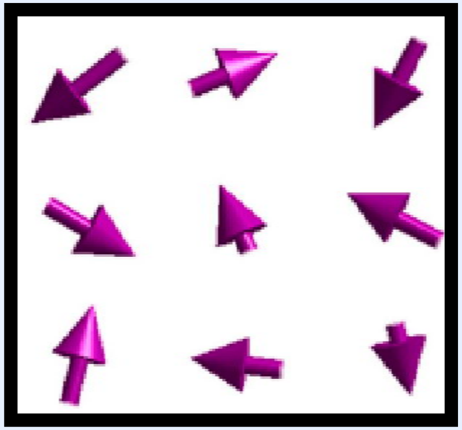

Vidro de spins

(Adaptado de S. Sato et al, Symmetry 12, 1910 (2020))

Ingredientes: desordem + frustração

PHYSICAL REVIEW B VOLUME 6, NUMBER 11 1 DECEMBER 1972

Magnetic Ordering in Gold-Iron Alloys*

V. Cannella†
Physics Department, Wayne State University, Detroit, Michigan 48202

and

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Institut für Festkörperforschung, Kernforschungsanlage, 517 *Jülich, Germany*
(Received 18 April 1972)

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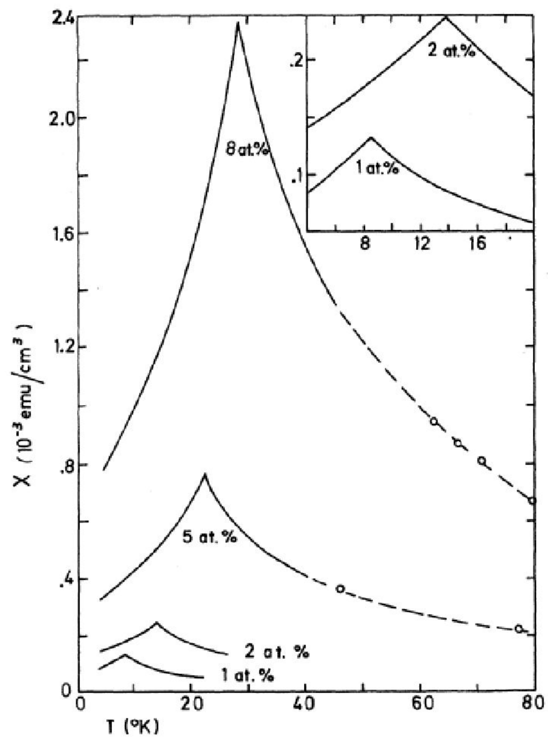

FIG. 9. Low-field susceptibility $\chi(T)$ for $1 \le C \le 8$ at.%. The data were taken every $\frac{1}{4}$°K in the region of the peak, and every $\frac{1}{2}$ or 1 °K elsewhere. The scatter is of the order of the thickness of the lines. The open circles indicate isolated points taken at higher temperatures.

Os primeiro vidros de spins eram metálicos.

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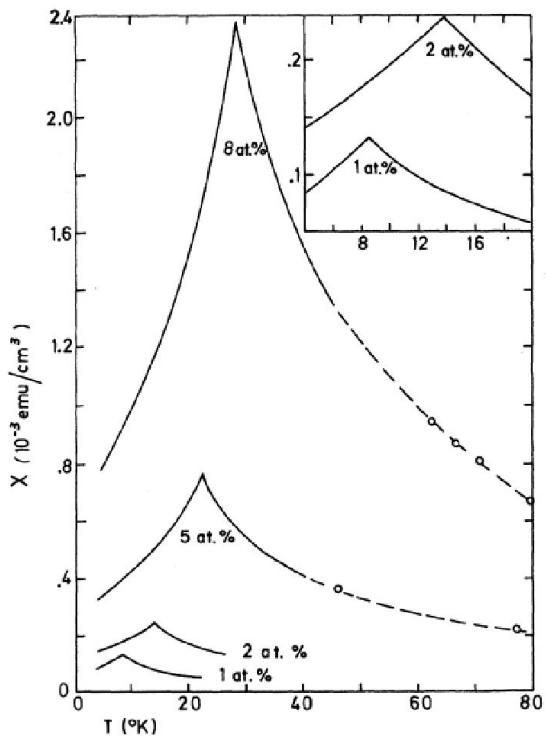

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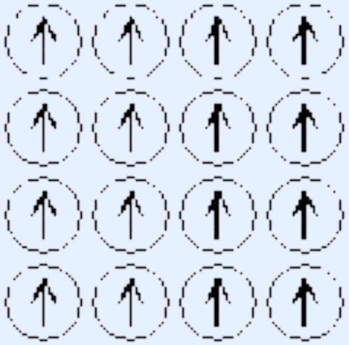
Os primeiro vidros de spins eram metálicos.

Como explicar?

Ordem + ausência de frustração:

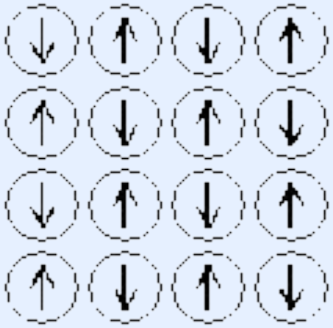
Ferromagnetismo ($T < T_c$):

$M \neq 0$



Antiferromagnetismo ($T < T_N$):

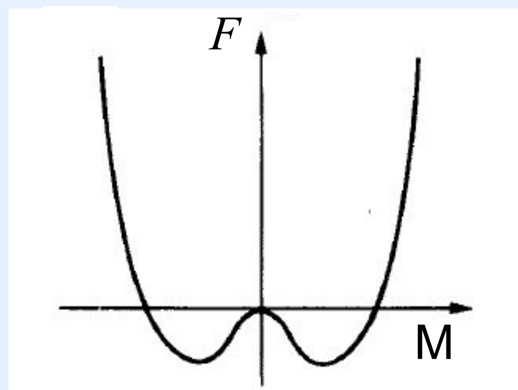
$M = 0$, mas $M_{sub\text{-}rede} \neq 0$

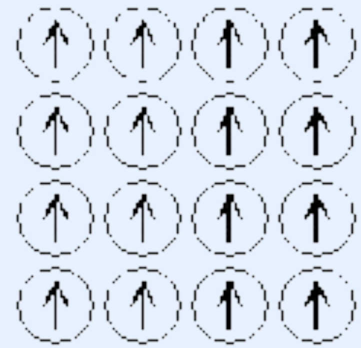


Ordem + ausência de frustração (T < T$_c$):

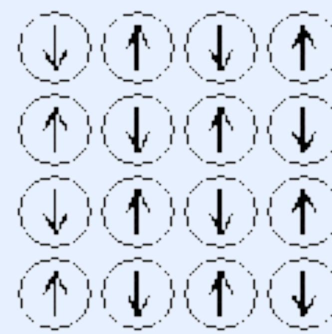
$$\mathcal{H} = \sum_{ij} J S_i S_j$$

FM: *J* < 0



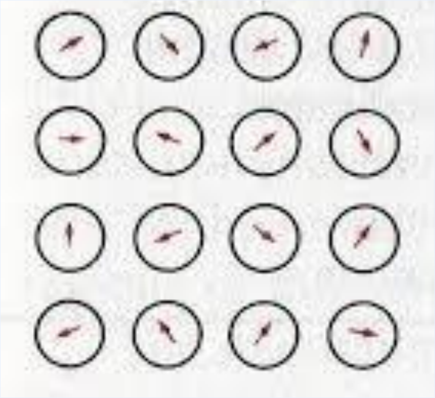


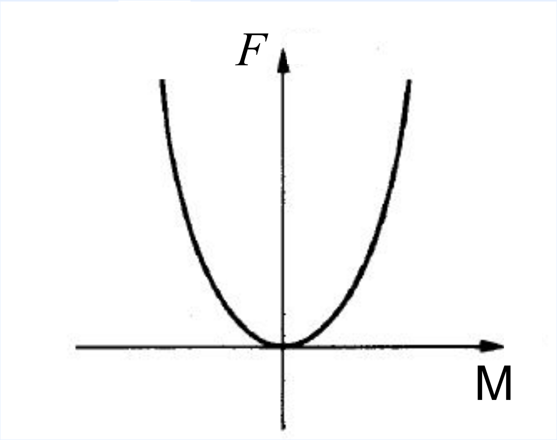
AF: *J* > 0



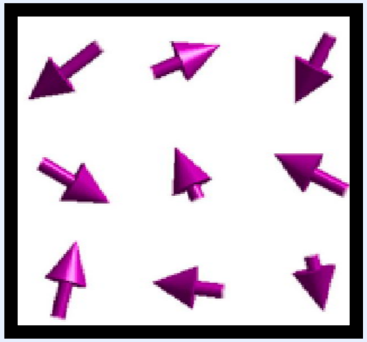
$T > T_c$: desordem térmica

Paramagnetismo: $J < 0$ ou $J > 0$, $M = 0$



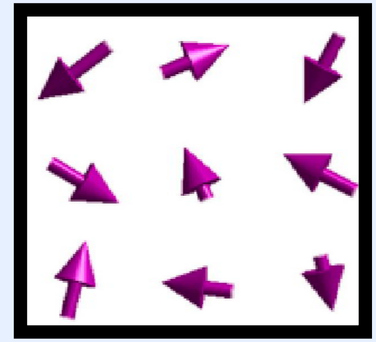


Desordem + frustração: Vidros de Spin



Desordem + frustração: Vidros de Spin

$$\mathcal{H} = \sum_{ij} J_{ij} S_i S_j$$



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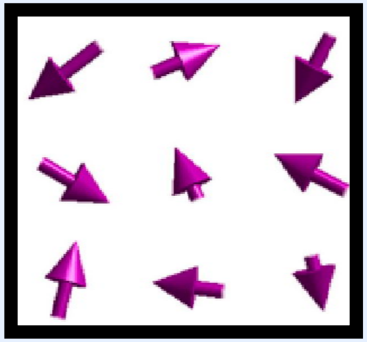
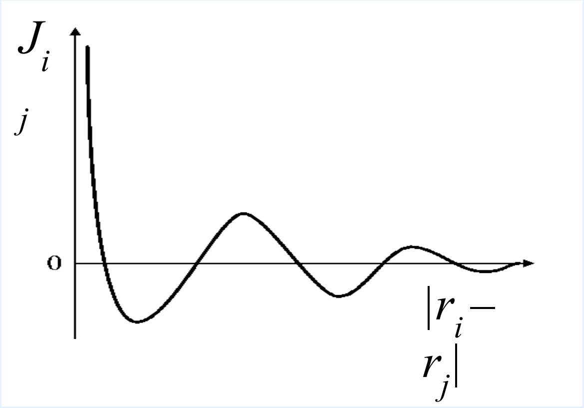
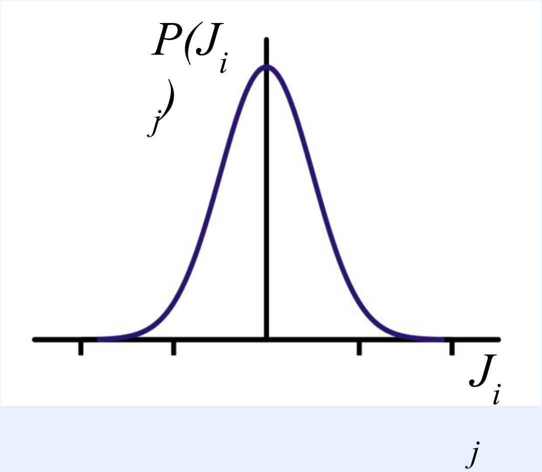


J_{ij}: RKKY, gaussiana, bimodal...





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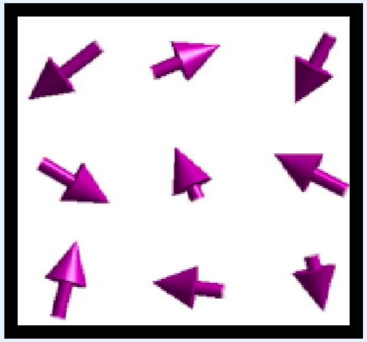

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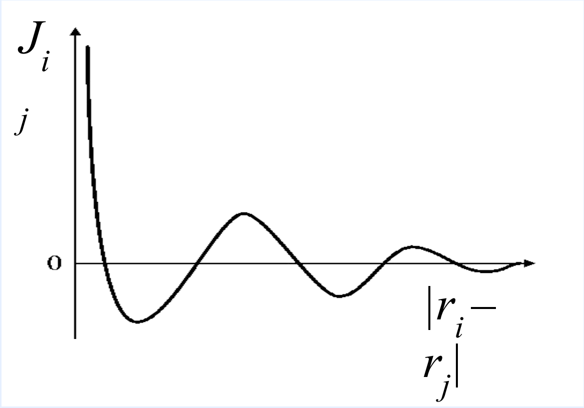

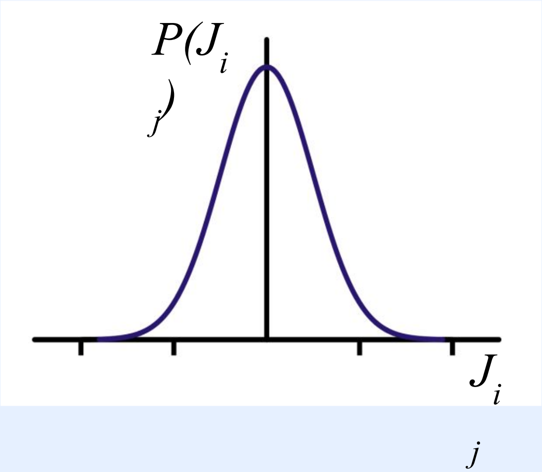

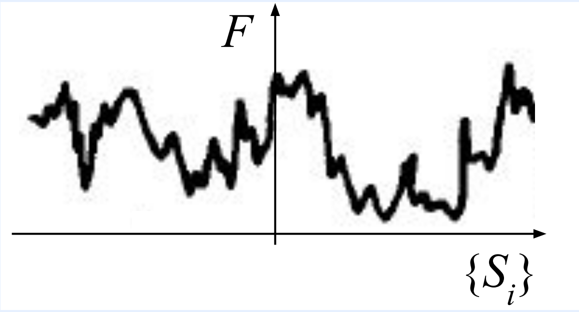

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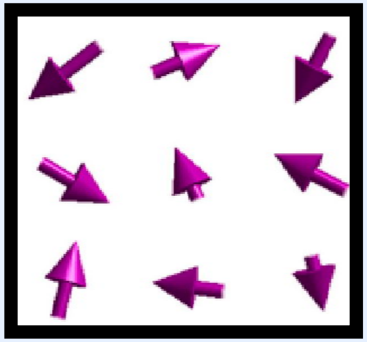

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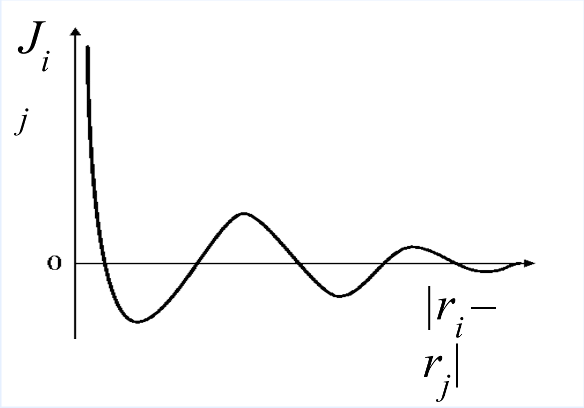

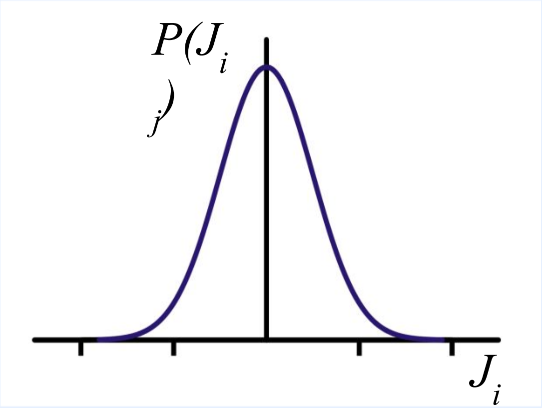

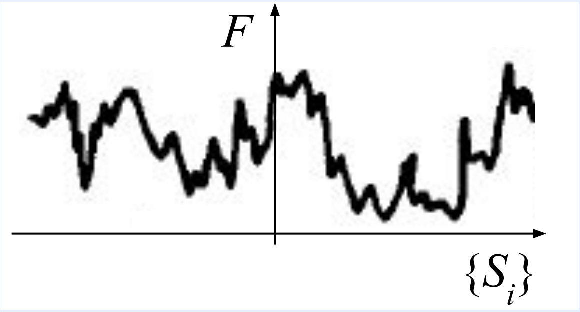

$T < T_f$: $M = 0$ & $M_{\text{sub-rede}} = 0$ ⇒ parâmetro de ordem?

Vidros de Spin: teoria

Edwards e Anderson (J Phys F 1975):

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} S_i S_j$$

J. Phys. F: Metal Phys., Vol. 5, May 1975. Printed in Great Britain. © 1975.

Theory of spin glasses

S F Edwards† and P W Anderson‡
Cavendish Laboratory, Cambridge, UK

Received 14 October 1974, in final form 13 February 1975

Parâmetro de ordem:

$$q = \lim_{t\to\infty} \frac{1}{N} \sum_i \overline{\langle S_i(0) S_i(t) \rangle} = \frac{1}{N} \sum_i \overline{\langle S_i \rangle^2}$$

Vidros de Spin: teoria

Edwards e Anderson (J Phys F 1975):

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} S_i S_j$$

Sherrington e Kirkpatrick (PRL 1975):

$$\mathcal{H} = \sum_{i,j} J_{ij} S_i S_j$$

$$\ln Z = \lim_{n \to 0} \frac{Z^n - 1}{n}$$

Campo médio e "replica trick" (n **réplicas** de $\mathcal{H}$):

VOLUME 35, NUMBER 26 PHYSICAL REVIEW LETTERS 29 DECEMBER 1975

Solvable Model of a Spin-Glass

David Sherrington* and Scott Kirkpatrick
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 16 October 1975)

Vidros de Spin: teoria

De Almeida e Thouless (J Phys A 1978): instabilidade da solução SK com campo magnético

J. Phys. A: Math. Gen., Vol. 11, No. 5, 1978. Printed in Great Britain

Stability of the Sherrington–Kirkpatrick solution of a spin glass model

J R L de Almeida†‡ and D J Thouless

Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT, UK

† On leave from Departmento de Fisica da U.F.Pe, Recife, Brasil.

‡ Partially supported by CNPq (Brazilian Government).

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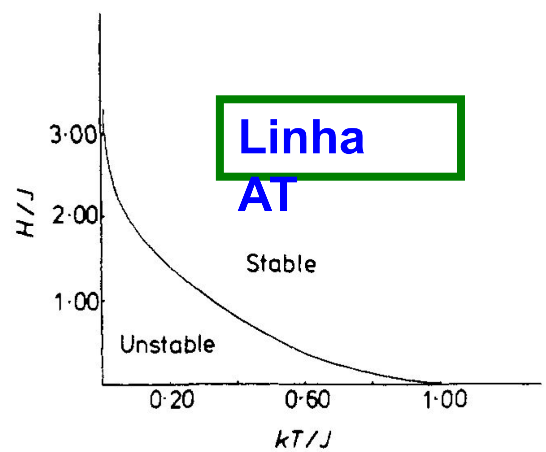

Figure 2. Phase diagram showing the limit of stability of the SK solution for the paramagnetic phase in the presence of a magnetic field H in the case $J_0 = 0$.

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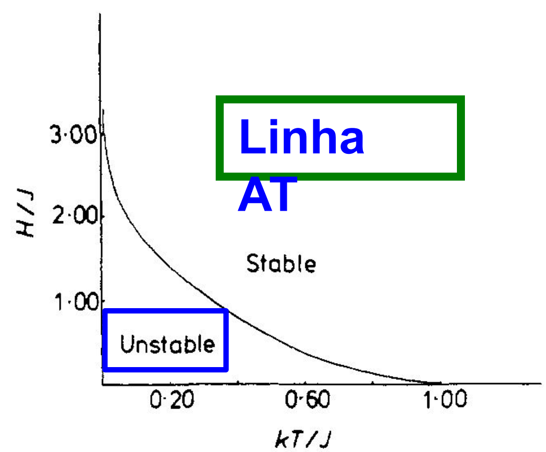

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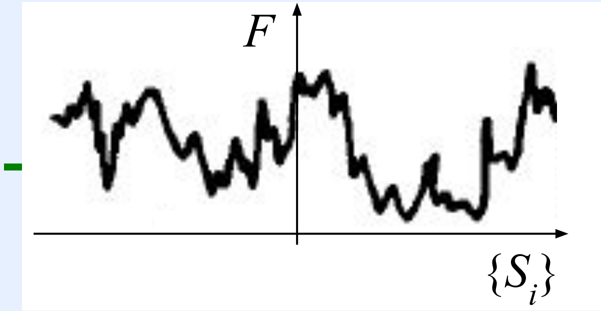
Parisi (PRL 1979): o modelo SK exibe fase VS com 'replica symmetry breaking' (RSB)



RSB: réplicas preparadas identicamen-te podem levar a medidas distintas

Volume 73A, number 3 PHYSICS LETTERS 17 September 1979

TOWARD A MEAN FIELD THEORY FOR SPIN GLASSES

G. PARISI
INFN, Laboratori Nazionali di Frascati, Italy

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Revised manuscript received 26 June 1979

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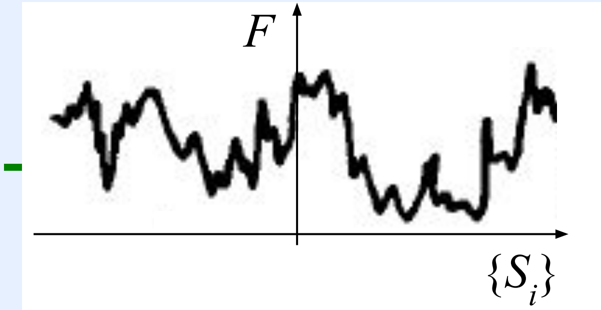

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Parisi overlap parameter:

$$q_{\alpha\beta} = \frac{1}{N}\sum_i S_{i\alpha}S_{i\beta}$$

$(\alpha, \beta) = \text{par de réplicas}$

Volume 73A, number 3 PHYSICS LETTERS 17 September 1979

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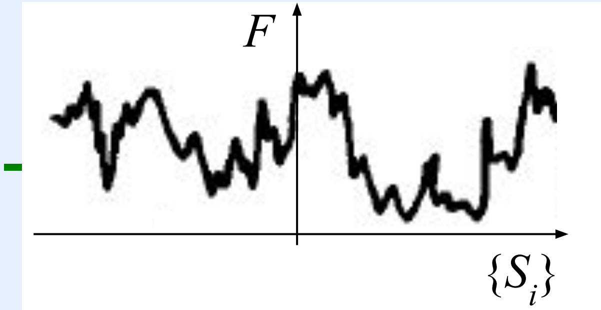

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VOLUME 43, NUMBER 23 PHYSICAL REVIEW LETTERS 3 DECEMBER 1979

Infinite Number of Order Parameters for Spin-Glasses

G. Parisi
Servizio Documentazione, Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, Frascati, Italy
(Received 22 June 1979)

Vidros de Spin: teoria

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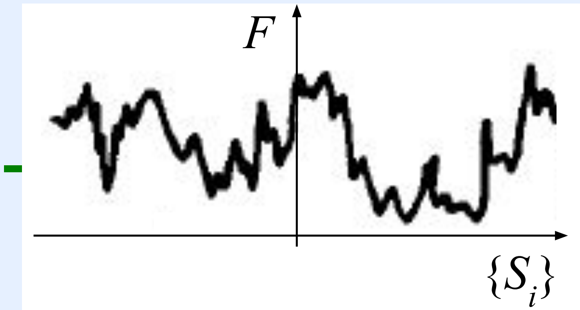

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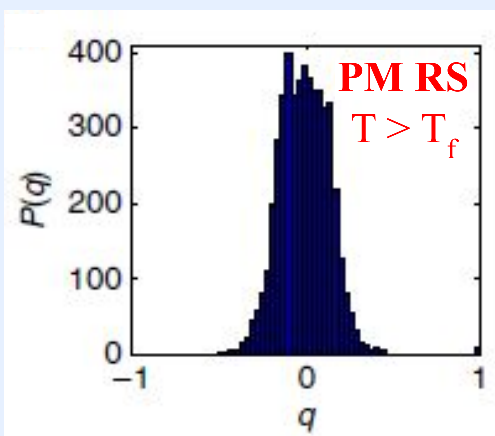

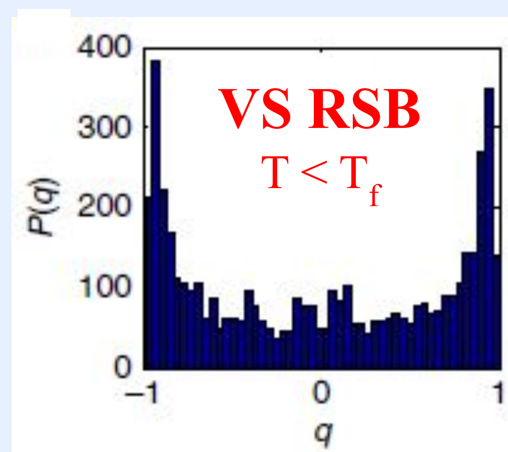

PHYSICAL REVIEW B VOLUME 6, NUMBER 11 1 DECEMBER 1972

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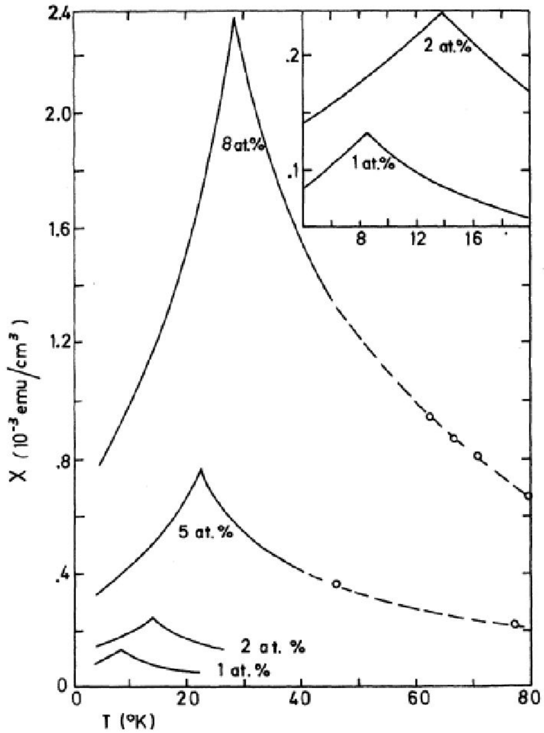

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Os primeiro vidros de spins eram metálicos.

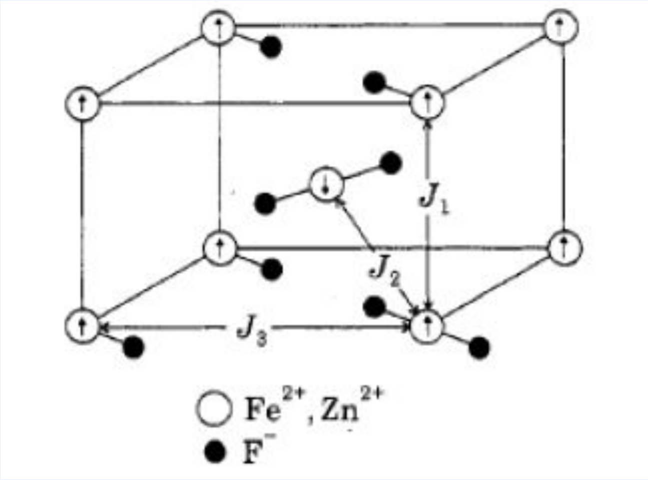
Um Vidro de Spins descoberto no DF-UFPE

(Montenegro, Rezende, Coutinho-Filho, J Appl Phys 1988)

$Fe_xZn_{1-x}F_2$ isolante

$x = 1 \rightarrow$ AF

$x = 0.25\ (\approx x_p) \rightarrow$ SG



3755 J. Appl. Phys. 63 (8), 15 April 1988 0021-8979/88/083755-03$02.40 © 1988 American Institute of Physics 3755

Evidence for a spin-glass behavior in the diluted antiferromagnet $Fe_xZn_{1-x}F_2$

F. C. Montenegro,[a] S. M. Rezende, and M. D. Coutinho-Filho
Universidade Federal de Pernambuco, Departamento de Fisica, 50.000 Recife, PE, Brazil

Termodinâmica, dinâmica, propriedades críticas.

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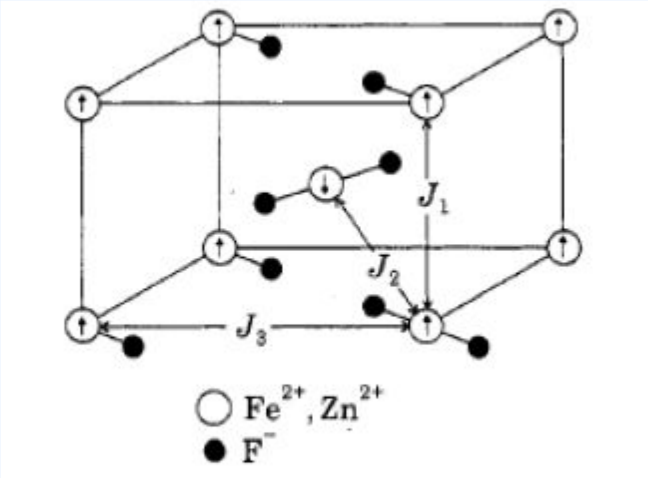

$$\mathcal{H} = \sum_{\langle i,i+\delta \rangle} J_{i,i+\delta}\varepsilon_i\varepsilon_{i+\delta}S_iS_{i+\delta}, \quad \delta = 1, 2, 3$$

VOLUME 91, NUMBER 19 — PHYSICAL REVIEW LETTERS — week ending 7 NOVEMBER 2003

Microscopic Description of an Ising Spin Glass near the Percolation Threshold

Paulo H. R. Barbosa,[1,2] E. P. Raposo,[1] and M. D. Coutinho-Filho[1]

[1]*Laboratório de Física Teórica e Computacional, Departamento de Física, Universidade Federal de Pernambuco, 50670-901, Recife-PE, Brazil*

[2]*Departamento de Física, Universidade Federal do Piauí, 64048-550, Teresina-PI, Brazil*

(Received 26 December 2002; published 6 November 2003)

Hamiltonianos dos VS magnéticos:

desordem + frustração

$$\mathcal{H} = \sum_{i_1 i_2} J^{(2)}_{i_1 i_2} S_{i_1} S_{i_2} + \frac{1}{2} \sum_{i_1 i_2 i_3 i_4} J^{(4)}_{i_1 i_2 i_3 i_4} S_{i_1} S_{i_2} S_{i_3} S_{i_4}$$

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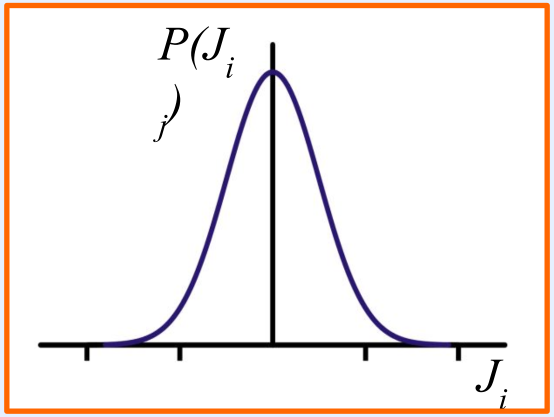
$J^{(2)}$, $J^{(4)}$: gaussianas



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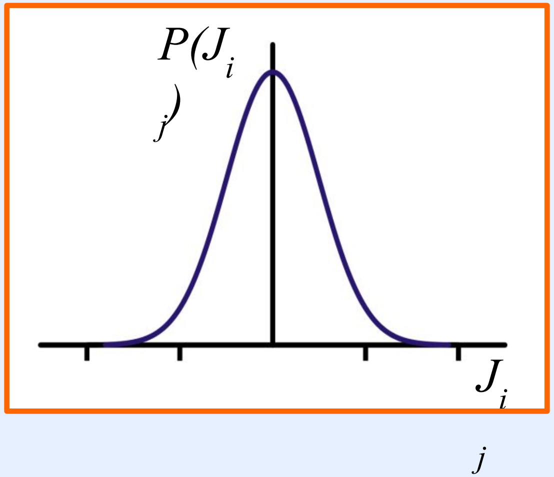
$J^{(2)}$, $J^{(4)}$: gaussianas

Modelo esférico p-spins

(p = 2+4)



Hamiltonianos dos VS magnéticos:

desordem + frustração

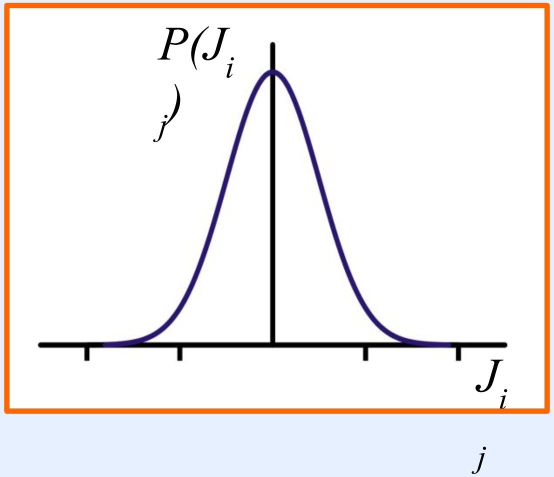
$$\mathcal{H} = \sum_{i_1 i_2} J^{(2)}_{i_1 i_2} S_{i_1} S_{i_2} + \frac{1}{2} \sum_{i_1 i_2 i_3 i_4} J^{(4)}_{i_1 i_2 i_3 i_4} S_{i_1} S_{i_2} S_{i_3} S_{i_4}$$

$J^{(2)}$, $J^{(4)}$: gaussianas

Modelo esférico p-spins

(p = 2+4)

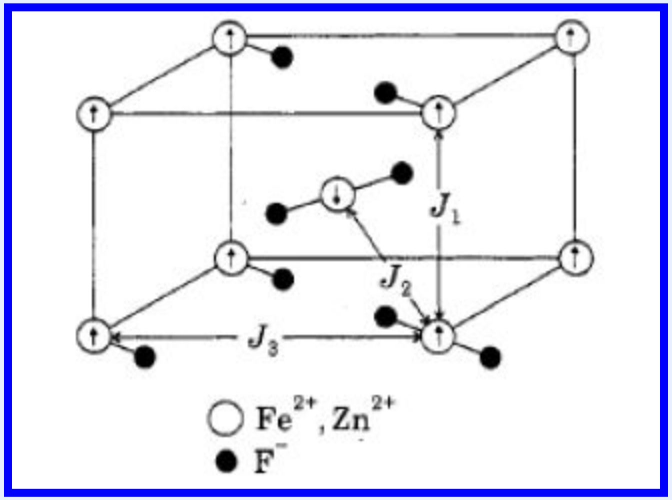
$$\sum_i S_i^2 = N$$



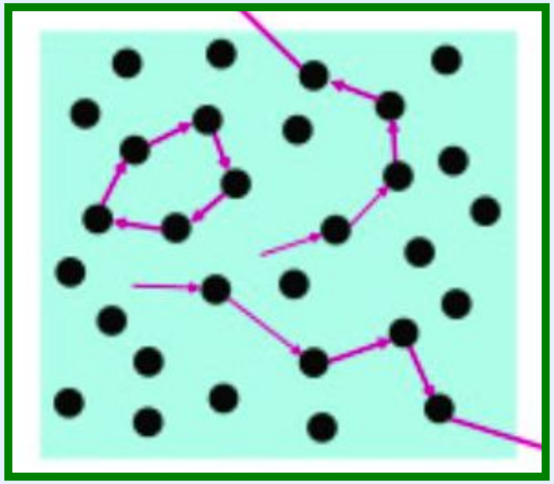
- Parte III: Aplicações em Física Estatística e Fotônica

Università di Roma “La Sapienza”

Giorgio Parisi



Vidros de spins magnéticos
(1970's)



Vidros de spins fotônicos
(2006...)

- Lasers convencionais $\leftrightarrow$ hamiltoniano

- Lasers convencionais $\leftrightarrow$ hamiltoniano
- Lasers aleatórios $\leftrightarrow$ desordem

- **Lasers convencionais ↔ hamiltoniano**

- **Lasers aleatórios ↔ desordem**

- **Lasers aleatórios: hamiltoniano com desordem?**

- **Lasers convencionais ↔ hamiltoniano**
- **Lasers aleatórios ↔ desordem**
- **Lasers aleatórios: hamiltoniano com desordem?**
- **Vidros de spins fotônicos?**

- Lasers convencionais ↔ hamiltoniano

- Lasers aleatórios ↔ desordem

- Lasers aleatórios: hamiltoniano com desordem?

- Vidros de spins fotônicos?

PRL 96, 065702 (2006) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2006

Glassy Behavior of Light

L. Angelani,[1] C. Conti,[2,3] G. Ruocco,[3,4] and F. Zamponi[3,4]
[1]*Research Center SMC INFM-CNR, c/o Università di Roma "La Sapienza," I-00185, Roma, Italy*
[2]*Centro Studi e Ricerche "Enrico Fermi," Via Panisperna 89/A, I-00184, Roma, Italy*
[3]*Research Center Soft INFM-CNR, c/o Università di Roma "La Sapienza," I-00185, Roma, Italy*
[4]*Dipartimento di Fisica, Università di Roma "La Sapienza," I-00185, Roma, Italy*
(Received 22 September 2005; published 16 February 2006)

- Lasers convencionais ↔ hamiltoniano

- Lasers aleatórios ↔ desordem

- Lasers aleatórios: hamiltoniano com desordem?

- Vidros de spins fotônicos?

PRL 96, 065702 (2006) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2006

Glassy Behavior of Light

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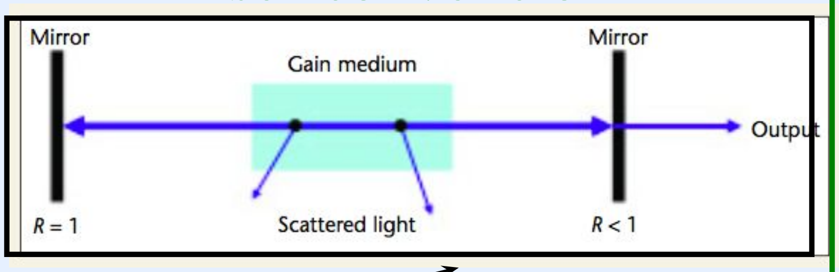
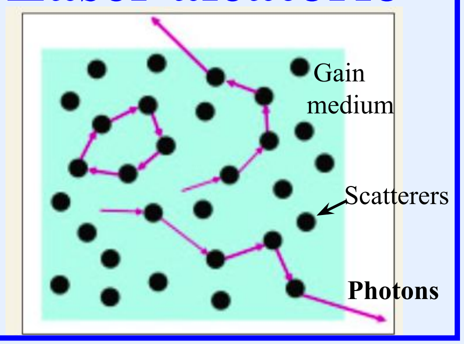
[4]*Dipartimento di Fisica, Università di Roma "La Sapienza," I-00185, Roma, Italy*
(Received 22 September 2005; published 16 February 2006)

Laser convencional

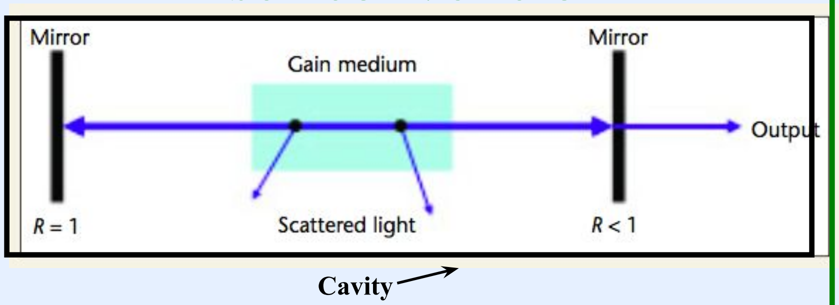


Laser aleatório

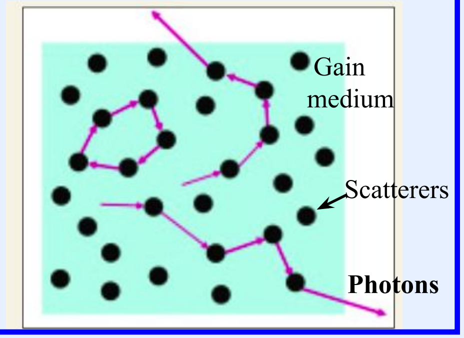


NL +
desordem

Laser convencional



Laser aleatório

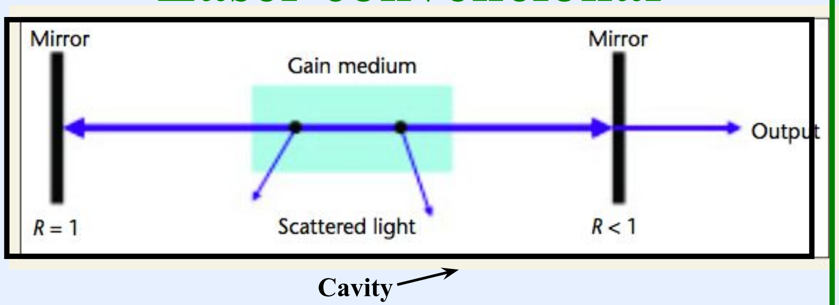


NL + desordem

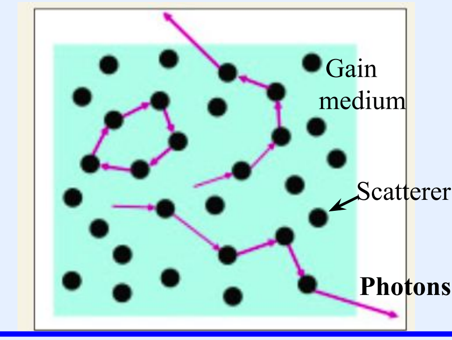
Hamiltoniano VS magnético:

$$\mathcal{H} = \sum_{i_1 i_2} J^{(2)}_{i_1 i_2} S_{i_1} S_{i_2} + \frac{1}{2} \sum_{i_1 i_2 i_3 i_4} J^{(4)}_{i_1 i_2 i_3 i_4} S_{i_1} S_{i_2} S_{i_3} S_{i_4}$$

Laser convencional



Laser aleatório



NL + **desordem**

Hamiltoniano VS magnético:

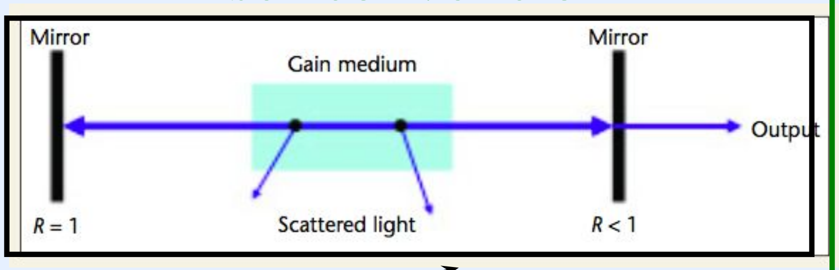
$$\mathcal{H} = \sum_{i_1 i_2} J^{(2)}_{i_1 i_2} S_{i_1} S_{i_2} + \frac{1}{2} \sum_{i_1 i_2 i_3 i_4} J^{(4)}_{i_1 i_2 i_3 i_4} S_{i_1} S_{i_2} S_{i_3} S_{i_4}$$

Hamiltoniano RL:

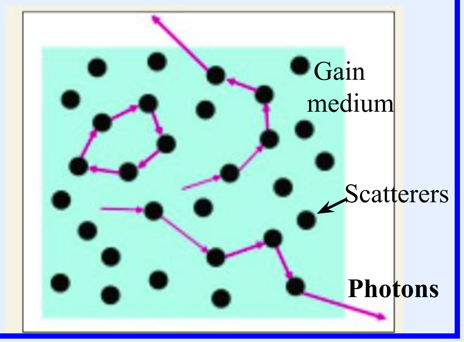
[Angelani et al, PRL (2006)...]

$$\mathcal{H} = \sum_{\{k_1 k_2\}'} g^{(2)}_{k_1 k_2} a_{k_1} a^*_{k_2} + \frac{1}{2} \sum_{\{k_1 k_2 k_3 k_4\}'} g^{(4)}_{k_1 k_2 k_3 k_4} a_{k_1} a^*_{k_2} a_{k_3} a^*_{k_4}$$

Laser convencional



Laser aleatório



NL + desordem

Hamiltoniano VS magnético:

$$\mathcal{H} = \sum_{i_1 i_2} J^{(2)}_{i_1 i_2} S_{i_1} S_{i_2} + \frac{1}{2} \sum_{i_1 i_2 i_3 i_4} J^{(4)}_{i_1 i_2 i_3 i_4} S_{i_1} S_{i_2} S_{i_3} S_{i_4}$$

Hamiltoniano RL:

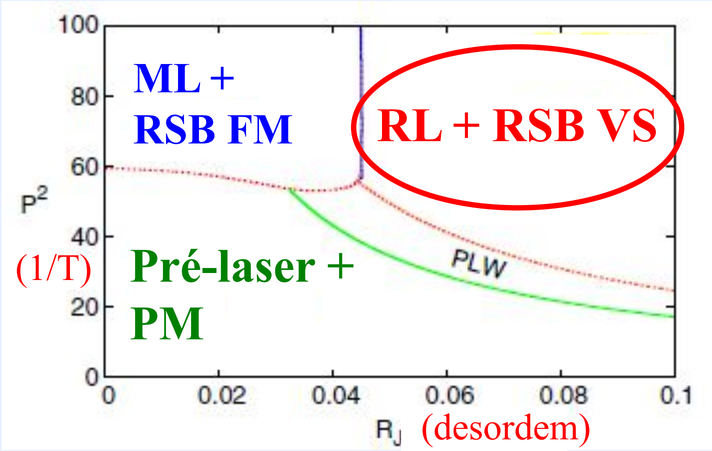
[Angelani et al, PRL (2006)...]

$g^{(2)}$, $g^{(4)}$: gaussianas

$$\mathcal{H} = \sum_{\{k_1 k_2\}'} g^{(2)}_{k_1 k_2} a_{k_1} a^*_{k_2} + \frac{1}{2} \sum_{\{k_1 k_2 k_3 k_4\}'} g^{(4)}_{k_1 k_2 k_3 k_4} a_{k_1} a^*_{k_2} a_{k_3} a^*_{k_4}$$

Diagramas de fases

- T (magnetismo) ↔ P^{-1} (fotônica)

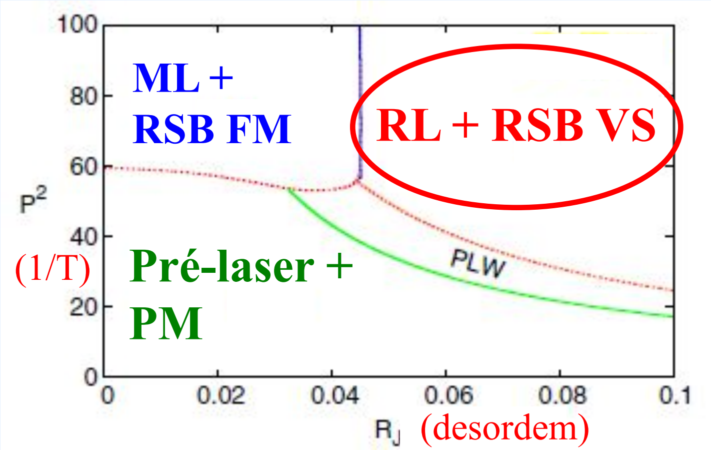


Diagramas de fases

- T (magnetismo) $\leftrightarrow$ P^{-1} (fotônica)
- grau de desordem [variâncias das P(J) e P(g)]
- grau de NL [razão entre variâncias $g^{(4)}/g^{(2)}$]

(i) T↑ / P↓: PM RS / **IW RS**

("all modes oscillate independently and incoherently")



(ii) T↓ / P↑ & fraca desordem:
FM / **ML** [RS ou RSB (random bond FM)]

("modes oscillate coherently with same phase in a ML lasing regime")

(iii) T↓ / P↑ & forte desordem: VS RSB / **RL RSB**

("synchronous oscillation is frustrated; modes do not oscillate coherently in a trivial way, i.e., with same phase, but are nontrivially correlated")

Teoria: $$q_{\alpha\beta} \propto \mathrm{Re}\{\sum_k a_{k\alpha} a^*_{k\beta}\}$$

Experimento: $$q_{\alpha\beta} \propto \sum_k \Delta_{k\alpha}\Delta_{k\beta}$$ $$\Delta_{k\alpha} = I_{k\alpha} - \bar{I}_k$$

Primeiro laser aleatório com fase VS (Ghofraniha et al, Nat Commun 2015): 2D T_5OC_x amorphous solid-state RL

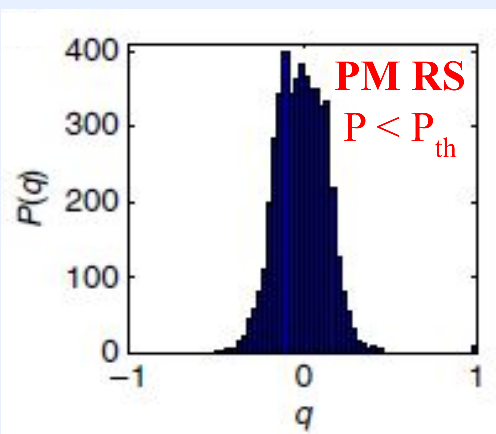
nature COMMUNICATIONS

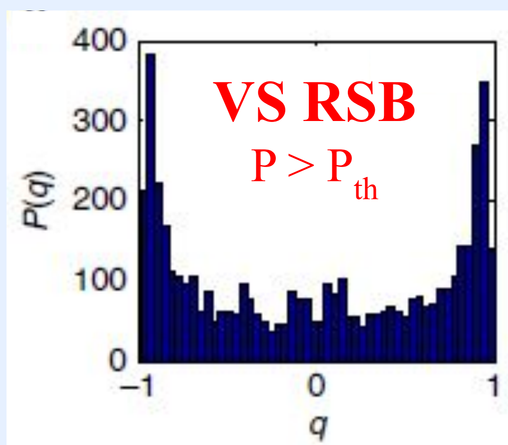
ARTICLE

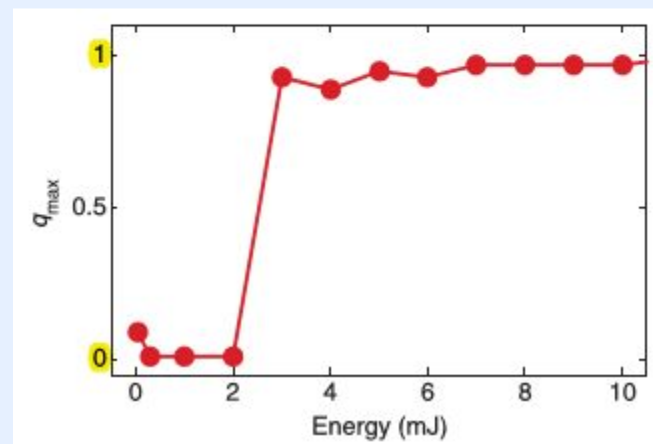
Received 18 Dec 2013 | Accepted 8 Dec 2014 | Published 14 Jan 2015 DOI: 10.1038/ncomms7058

Experimental evidence of replica symmetry breaking in random lasers

N. Ghofraniha[1,2,3], I. Viola[2,4], F. Di Maria[5,6], G. Barbarella[5], G. Gigli[4,7], L. Leuzzi[1,2] & C. Conti[2,3]







Segundo laser aleatório com fase VS (Gomes et al, Sci Rep 2016): 3D Nd^{+3}-doped YBO_3

SCIENTIFIC REPORTS

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Received: 02 March 2016
Accepted: 26 May 2016
Published: 13 June 2016

Observation of Lévy distribution and replica symmetry breaking in random lasers from a single set of measurements

Anderson S. L. Gomes[1], Ernesto P. Raposo[2], André L. Moura[1,3], Serge I. Fewo[4], Pablo I. R. Pincheira[1], Vladimir Jerez[5], Lauro J. Q. Maia[6] & Cid B. de Araújo[1]

PRL 119, 163902 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

Replica Symmetry Breaking in the Photonic Ferromagneticlike Spontaneous Mode-Locking Phase of a Multimode Nd:YAG Laser

André L. Moura,[1,2] Pablo I. R. Pincheira,[1] Albert S. Reyna,[1] Ernesto P. Raposo,[3,*] Anderson S. L. Gomes,[1] and Cid B. de Araújo[1]

PHYSICAL REVIEW LETTERS 122, 143903 (2019)

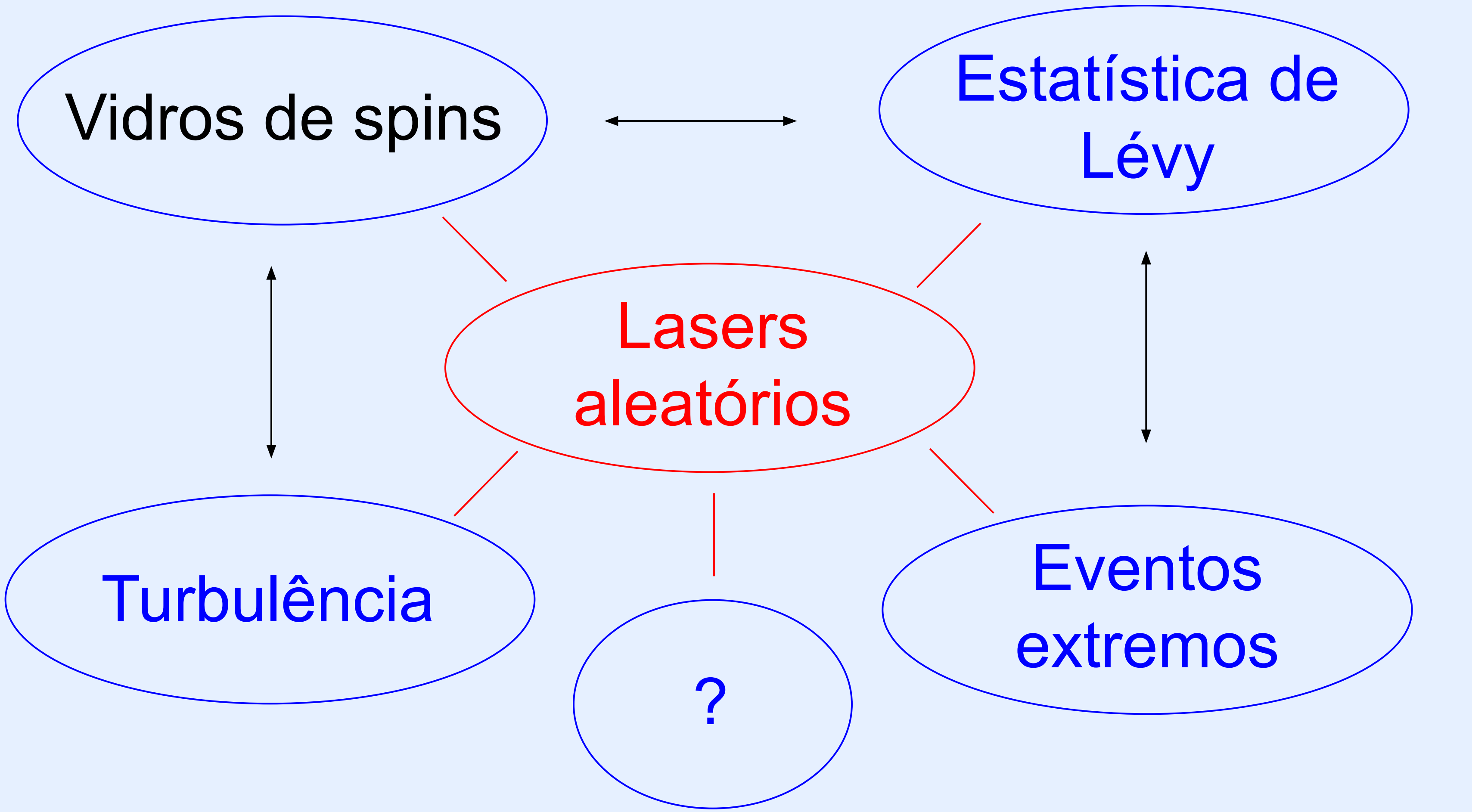
Evidence of a Floquet Phase in a Photonic System

Ernesto P. Raposo,[1] Iván R. R. González,[1] A. M. S. Macêdo,[1] Bismarck C. Lima,[2] Raman Kashyap,[3] Leonardo de S. Menezes,[2] and Anderson S. L. Gomes[2]

PHYSICAL REVIEW LETTERS 132, 093801 (2024)

Observation of Replica Symmetry Breaking in Standard Mode-Locked Fiber Laser

Nicolas P. Alves,[1] Wesley F. Alves,[1] André C. A. Siqueira,[1] Naudson L. L. Matias,[1] Anderson S. L. Gomes,[1] Ernesto P. Raposo,[2] and Marcio H. G. de Miranda[1]


Vidros de spins
Estatística de Lévy
Lasers aleatórios
Turbulência
?
Eventos extremos


SCIENTIFIC REPORTS
OPEN
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PHYSICAL REVIEW A 91, 043827 (2015)
Analytical solution for the Lévy-like steady-state distribution of intensities in random lasers
E. P. Raposo*
Laboratório de Física Teórica e Computacional, Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife-PE, Brazil
A. S. L. Gomes†
Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife-PE, Brazil
(Received 2 February 2015; published 16 April 2015)
Vidros de spins
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nature COMMUNICATIONS
Received 25 Jan 2017 | Accepted 24 Apr 2017 | Published 31 May 2017
DOI: 10.1038/ncomms15731
OPEN
Turbulence hierarchy in a random fibre laser
Iván R. Roa González[1], Bismarck C. Lima[2], Pablo I.R. Pincheira[2], Arthur A. Brum[1], Antônio M.S. Macêdo[1], Giovani L. Vasconcelos[1], Leonardo de S. Menezes[2], Ernesto P. Raposo[1], Anderson S.L. Gomes[2] & Raman Kashyap[3,4]
PHYSICAL REVIEW A 96, 013834 (2017)
Extreme-value statistics of intensities in a cw-pumped random fiber laser
Bismarck C. Lima,[1] Pablo I. R. Pincheira,[1] Ernesto P. Raposo,[2,*] Leonardo de S. Menezes,[1] Cid B. de Araújo,[1] Anderson S. L. Gomes,[1] and Raman Kashyap[3]

Primeira demonstração de turbulência e vidro de spin no mesmo sistema

4 transições simultâneas: pré-laser/RL, Gauss/Lévy, normal/turbulento, PM/VS

SCIENTIFIC REPORTS

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Coexistence of turbulence-like and glassy behaviours in a photonic system

Received: 26 July 2018
Accepted: 5 November 2018
Published online: 19 November 2018

Iván R. R. González[1], Ernesto P. Raposo[1], Antônio M. S. Macêdo[1], Leonardo de S. Menezes[2] & Anderson S. L. Gomes[2]

Coeficiente de correlação de Pearson: ambas as fases

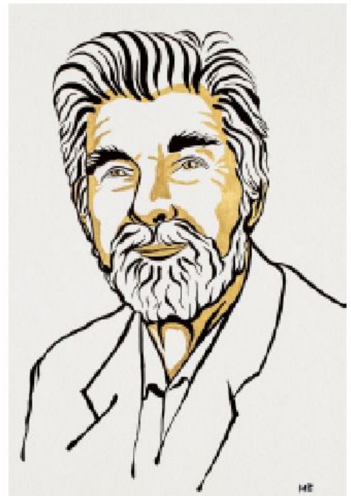
$$Q_{\alpha\beta,\tau} = \frac{\sum_k \delta_{\alpha\tau}(k)\delta_{\beta\tau}(k)}{\sqrt{\sum_k \delta^2_{\alpha\tau}(k)}\sqrt{\sum_k \delta^2_{\beta\tau}(k)}}.$$

The Nobel Prize in Physics 2021

Ill. Niklas Elmehed © Nobel Prize Outreach

Syukuro Manabe

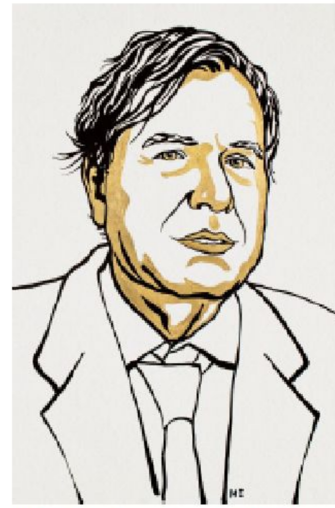
Prize share: 1/4



Ill. Niklas Elmehed © Nobel Prize Outreach

Klaus Hasselmann

Prize share: 1/4



Ill. Niklas Elmehed © Nobel Prize Outreach

Giorgio Parisi

Prize share: 1/2

The Nobel Prize in Physics 2021 was awarded "for groundbreaking contributions to our understanding of complex systems" with one half jointly to Syukuro Manabe and Klaus Hasselmann "for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming" and the other half to Giorgio Parisi "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales."

KUNGL.
VETENSKAPS-
AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

5 OCTOBER 2021

Scientific Background on the Nobel Prize in Physics 2021

"FOR GROUNDBREAKING CONTRIBUTIONS TO OUR UNDERSTANDING OF COMPLEX PHYSICAL SYSTEMS"

The Nobel Committee for Physics

www.nobelprize.org/uploads/2021/10/sciback_fy_en_21.pdf

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J. Phys. A: Math. Gen., Vol. 11, No. 5, 1978. Printed in Great Britain

Stability of the Sherrington–Kirkpatrick solution of a spin glass model

J R L de Almeida†‡ and D J Thouless

Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT, UK

† On leave from Departmento de Fisica da U.F.Pe, Recife, Brasil.

‡ Partially supported by CNPq (Brazilian Government).

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Observation of Lévy distribution and replica symmetry breaking in random lasers from a single set of measurements

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PRL 119, 163902 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

Replica Symmetry Breaking in the Photonic Ferromagneticlike Spontaneous Mode-Locking Phase of a Multimode Nd:YAG Laser

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[1]*Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife, Pernambuco, Brazil*
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(Received 20 November 2016; revised manuscript received 12 July 2017; published 20 October 2017)

SCIENTIFIC REPORTS

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KUNGL. VETENSKAPS-AKADEMIEN
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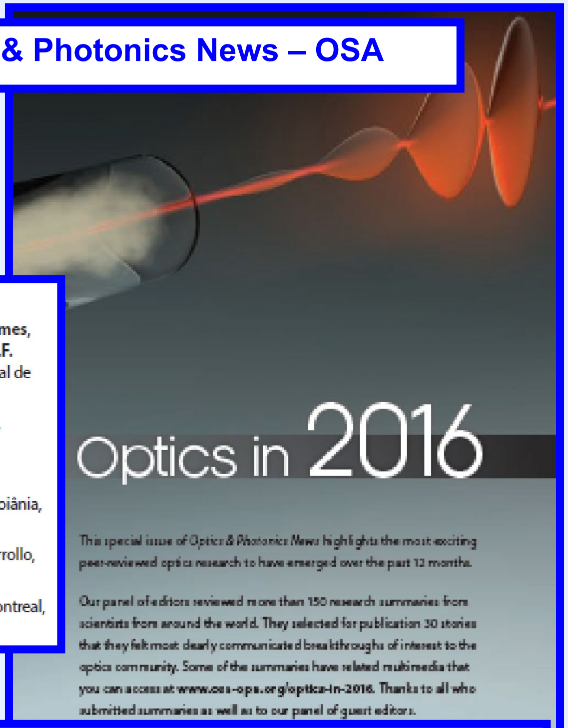
photorefractive disordered waveguides [90]. Finally, the nature of the random laser system allows for the concomitant observation of replica symmetry breaking and connection between spin-glasses and turbulence [34], particularly nonlinear wave interactions, which link the early work of Hasselmann [40, 41] to that of Parisi and to the role of disorder and fluctuations in complex systems in general.

Optics & Photonics News – OSA



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E.P. Raposo (ernesto@df.ufpe.br), **A.S.L. Gomes**, **P.I.R. Pincheira**, **B.C. Lima**, **S.J.M. Carreño**, **A.F. Silva** and **C.B. de Araújo**, Universidade Federal de Pernambuco, Recife, Brazil

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V. Jerez, Universidad de Investigación y Desarrollo, Bucaramanga, Colombia

M. Gagné and **R. Kashyap**, Polytechnique Montreal, Canada

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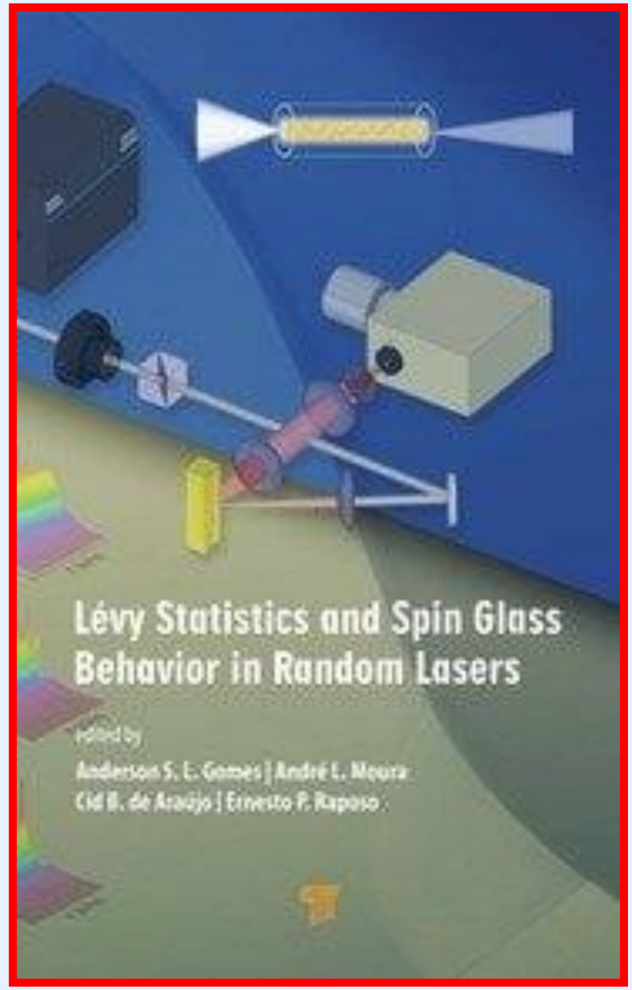

Conclusões

- Física Estatística aplicada a caminhadas aleatórias de buscas eficientes e lasers aleatórios: distribuições de Lévy e vidros de spins.

Agradecimentos: colaboradores, CNPq, CAPES, FACEPE.

Obrigado!

Turbulência em lasers aleatórios

nature COMMUNICATIONS
Received 25 Jan 2017 | Accepted 24 Apr 2017 | Published 31 May 2017
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Turbulence hierarchy in a random fibre laser

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Fluidos: dist incrementos de velocidades $P(\delta v)$

Fotônica: dist incrementos de intensidades $P(\delta I)$

→

- **Gaussiana** em grandes separações espaciais / temporais
- **Não-gaussiana** (heavy-tail) em pequenas separações

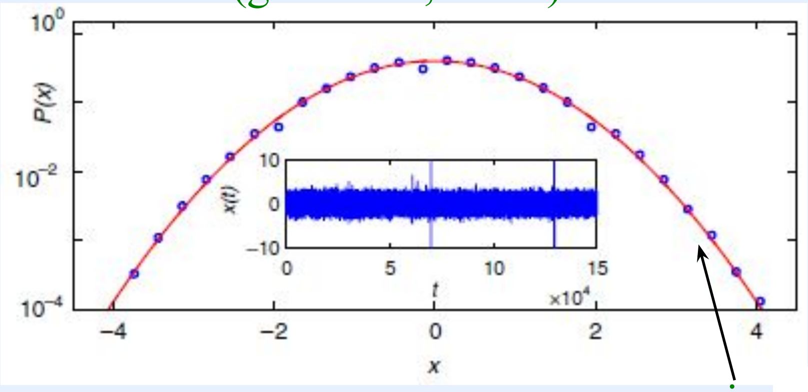
- **Turbulência em fluidos** (Kolmogorov,1940's):
  - Escalas (hierarquias) de energia (eddies)
  - Fluxo ('cascata') de energia entre escalas
  - Flutuações no fluxo (intermitência)

- **Nosso modelo:**
  ($x \equiv \delta I$)

$$P_N(x) = \frac{\omega^{1/2}}{\sqrt{2\pi\varepsilon_0}\,\Gamma(\boldsymbol{\beta})} G^{N+1,0}_{0,N+1}\left(\begin{matrix} - \\ \boldsymbol{\beta} - 1/2, 0 \end{matrix} \,\middle|\, \frac{\omega x^2}{2\varepsilon_0} \right)$$

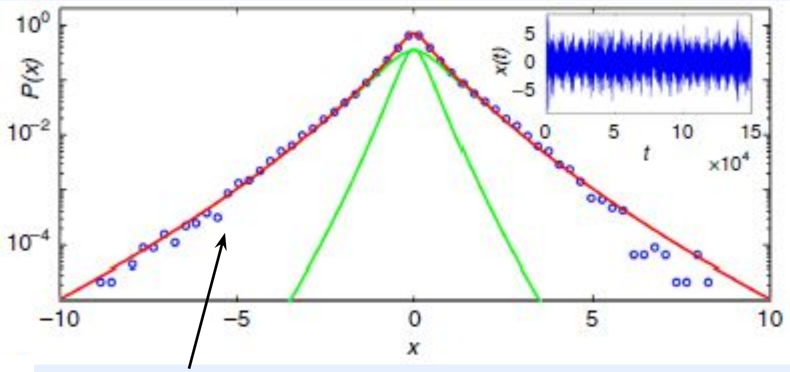
Excelente concordância teoria/experimento (Er-doped random fiber laser):

$P < P_{th}$: fase pré-laser não turbulenta
(gaussiana, $N = 0$)



gaussiana

$P > P_{th}$: fase RL turbulenta
(stretched exp, $N = 6$)



heavy tails